CHILDREN'S MEANING-MAKING ACTIVITY WITH DYNAMIC MULTIPLE REPRESENTATIONS IN A PROBABILITY MICROWORLD

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This paper describes how fourth grade children used representations in a computer microworld as both *objects to display* and interpret data, and as *dynamic objects of analysis* during experimentation. The children used the multiple representations in both static and dynamic form to develop their probabilistic reasoning and explore concepts such as the law of large of numbers. The results are from a 6-week teaching experiment investigating children's development of probabilistic reasoning with a newly developed probability microworld.

Design of the *Probability Explorer* Microworld

One of the most promising uses of computer technology in mathematics education is the ability to view multiple representations of phenomenon. Many software applications for middle and secondary school students include dynamic multiple representations. Although several software packages exist that allow elementary students to represent data in multiple forms, the representations are often only used as *objects of display* rather than *dynamic objects of analysis*.

My intent was to create a microworld containing multi-linked representations that update simultaneously as random events are simulated. The representations available in the *Probability Explorer* microworld at the time of the study included: 1) moveable iconic representations of every trial; 2)"stacking columns" to create pictographs; 3) a data table displaying results as a count, fraction, decimal and percent; 4) a pie graph; and 5) a bar graph.

Theoretical Framework

The theoretical foundation for the software design, instructional approach, and methods of inquiry is based on a constructivist theory of learning and research on students' use of microworlds (Steffe & Wiegel, 1994; Battista, 1998; Olive, 1994). I believe the tools available in a computer environment, meaningful instructional and playful activities, students' understanding, and their social and computer interactions all operate *interactively* as potential meaning-making agents for students' construction of concepts. Children interacting in a "fertile computer environment" (Battista, 1998) can learn through developing theories-in-action as they generate intuitive-based theories and modify them as they reflect upon experiences that either confirm their intuitions or challenge their theory through perturbations (Land & Hannafin, 1996).

Methods of Inquiry

The author conducted a 6-week (12 hours) constructivist teaching experiment (Steffe, 1991), including pre and post task-based interviews, with three fourth grade children. The purpose of the study was two-fold: 1) to study how children develop probabilistic reasoning in a dynamic multi-representational environment; and 2) to gather evidence of how children used microworld tools to inform future development of the software and instructional activities.

Each teaching session was video and audio taped to capture group interactions and a PC-to-TV converter was used to video children's computer interactions. The author was the lead teacher-researcher (T-R) while another T-R facilitated activities and a non-participant observer kept written records. After each session, the author critically reviewed the videotapes and developed subsequent tasks in the children's zone of potential construction (Olive, 1994).

The tasks included simulations with both equiprobable (e.g., coin flips, die toss) and unequiprobable (e.g., marble bag containing 1 white and 3 black marbles) events. In addition, the children also used a "weighting" tool in which the probability of an event could be changed (e.g., changing the likelihood of heads and tails for an unfair coin). The main tasks were planned and posed by the lead T-R, but many tasks were created on-the-fly based on the children's interactions, and "what if" or "can I try" questions. After completion of the teaching experiment, all sessions were transcribed and annotated while watching each videotape. The initial annotation included social and computer interactions and preliminary conjectures of children's meaning-making. Subsequent analysis revealed meaning-making themes for each child.

Results

One theme was the children's investigation of what Carmella termed as the "evening out" phenomenon (EOP). Although the children used many of the representations as *objects to display the data*, their observations during the simulation process facilitated their use of the representations as *dynamic objects of analysis*. It was the dynamic process that engaged students in exploring and developing their understanding of the EOP (i.e., law of large numbers).

The children used all the representations available in the microworld to re-present and make sense of randomly generated data after a simulation was complete (representations as *objects of display*). They often discussed the range of results by comparing the "highest" number of outcomes to the "smallest" number in the stacking columns. They also used the graphs and data table to make sense of the data for both a small and large number of trials.

During the first teaching session, the children discovered they could display the graphs during a simulation and watch the graphs do the "hula." This led them to spontaneously run a large number of trials and comment on the distribution of data during the simulation process (representations as *dynamic objects of analysis*). They discussed that results were not always as they expected with a small number of trials, but the pie graph "would hardly move at all" and "even out" (equiprobable outcomes) in a large number of trials.

Example Investigation

During the third teaching session, the children investigated a with-replacement experiment with a bag of 2 white and 2 black marbles (Figure 1a). For 10 trials, the children predicted many "5 and 5's" and "6 and 4's," ran several trials of 10, stacked their results, and viewed the pie graph and table after each simulation. When Amanda got 9 white and 1 black marble (Figure 1b), she squealed “oh my, this is very unlikely, usually you get 6 and 4, 5 and 5 and usually they are pretty much the same number.” Carmella and Jasmine gathered around Amanda’s screen, observed the representations as *objects of display*, commented on how different the 9-1 result looked from results of 5-5, 6-4 and 7-3.

Carmella and Jasmine jumped to running 500 trials “to see if the pie will even out.” Based on their prior experiences with the dynamic simulation process, they anticipated the “even out” results. Amanda ran a large number of trials and predicted “I think maybe they [the number of black and white marbles] are going to be far apart at one point and then get very close.” Carmella and Jasmine watched their graph during the simulation and noticed the large variation in the beginning (more black than white marbles in 50 trials) and how the “moon [white] made a comeback” and “evened out” as the trials approached 500 (Figure 1c).

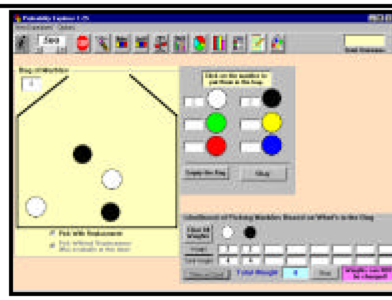

Figure 1a. Equiprobable marble bag experiment

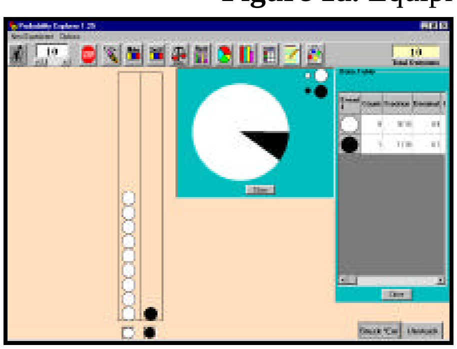

Figure 1b. Amanda’s “unlikely” result

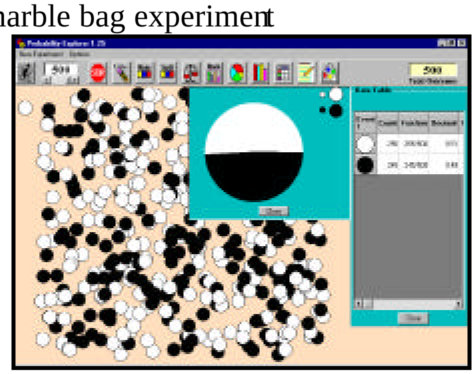

Figure 1c. Tendency towards “even”

Figure 1. Screenshots from marble bag experiment

In this episode, the children used the multiple representations as both *objects of display* and *dynamic objects of analysis*. Their use of representations as *objects of display* allowed them to critically analyze the results and make connections between the representations. However, their analysis of the representations during simulations, and their prediction based on prior dynamic visualizations, demonstrates how the use of representations as *dynamic objects of analysis* can facilitate their reasoning about powerful probabilistic concepts.

Children's Individual Use of Static and Dynamic Representations

The dynamic link between numerical and graphical results during the simulation process facilitated Carmella's theory-in-action about the EOP. After her initial experiences with equiprobable situations, Carmella used the representations to understand that a large number of trials would not result in an "even" distribution of outcomes, but rather, should approach a distribution close to the "weights" (i.e., theoretical probability) in unequiprobable situations. In addition, her use of representations as both *objects of display* and *dynamic objects of analysis* helped her think about the deviation from the "expected" based on theoretical probability. For example, with 10 trials of a fair coin toss, she considered a deviation of two (from the expected 5-5) greater than the same deviation with 100 trials (48-52). The iconic stacked data representation and the numerical displays in the data table focused her on the absolute variability in the range with a small number of trials. However, the dynamic pie graph facilitated her recognition that the important visual variability in the range was relative to the total number of trials (e.g., "the slices stay mostly the same" as the number of trials increased).

Jasmine used all the representations available in the microworld as *objects of display*. She once commented that she knew "five ways to tell" what the results were from an experiment (data table, pie graph, bar graph, stacking columns, and manually sorting and counting). From her first experience with the representations as *dynamic objects of analysis,* she made clear and accurate connections between the motion of the graphs, the random simulation process, updated results, and the tendency towards “settling down” near an even distribution (for equiprobable outcomes). The pie graph representation was a major cognitive prompt for Jasmine in a variety of contexts and induced perturbations that helped her understand the EOP and part-whole relationships. Her use of the pie graph as a *dynamic object of analysis* helped develop her understanding of the EOP similar to Carmella’s. In addition, she also used the pie graph as an *object of display* to help her transition to part-whole reasoning by thinking about the whole pie as a set number and then reasoning about the number associated with each slice relative to the whole (e.g., 1/10 is bigger than 1/20 because a pie cut in 10 has bigger slices).

Amanda’s use of the representations as *objects of display* was limited by her weak conceptual understanding of numbers and graphs. Her reasoning with the representations was not always probabilistic (e.g., she often focused on the “bumpiness” of the lines separating pie segments rather than the relative size of the slices). Amanda never enacted a theory-in-action about the EOP like the other children. Although she made reference to the pie graph “staying in the same place” during a simulation with a large number of trials, she made only occasional references to the actual simulation process or the theoretical probability in her analysis of the *dynamic objects*. She rarely reflected on the relationships between multiple representations and between empirical results and theoretical probability. Thus, for Amanda, the representations, as static and dynamic objects, were often a deterrent in her development of probabilistic reasoning.

Conclusion

As shown with Carmella and Jasmine, dynamically linked multiple representations have the *potential* to facilitate children’s probabilistic reasoning and encourage them to develop theories-in-action about the law of large numbers. The representations were mostly not helpful for Amanda. Thus, I added a feature in the microworld to allow the user to change the speed of a simulation. Slowing down the simulation process should allow students to attend to the changes in the representations and reflect on the effect of the number of trials on the results, rather than just watching the dynamic “motion.”

Using *Probability Explorer* can substantially extend typical experiences with physical objects and lead children to play, experiment, predict, and discover probabilistic ideas by using multiple representations as both static and dynamic objects. This study begins the research on children’s use of dynamic representations in developing probabilistic reasoning.

References

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